

Study of the de Almeida-Thouless line using one-dimensional power-law diluted Heisenberg Spin Glasses

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We test for the presence or absence of the de Almeida-Thouless line using one-dimensional power-law diluted Heisenberg spin glass model, in which the rms strength of the interactions decays with distance, r as $1/r^\sigma$. It is argued that varying the power σ is analogous to varying the space dimension d in a short-range model. For $\sigma = 0.6$, which is in the mean field regime, we find clear evidence for an AT line. For $\sigma = 0.85$, which is in the non-mean-field regime and corresponds to a space dimension of close to 3, we find no AT line, though we cannot rule one out for very small fields. Finally for $\sigma = 0.75$, which is in the non-mean-field regime but closer to the mean-field boundary, the evidence suggests that there is an AT line, though the possibility that even larger sizes are needed to see the asymptotic behavior can not be ruled out.

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I. INTRODUCTION

One of the major unsolved questions in the field of spin glasses is the nature of the low-temperature glassy phase. Two main candidate theories exist: the replica symmetry breaking (RSB) theory which assumes that real spin glasses behave in a similar way to the mean-field theory¹, and the droplet model^{2,3}, which provides a phenomenological approach. An important difference between the predictions of the two approaches is whether or not there is a line of phase transitions, known as the de Almeida-Thouless⁴ (AT) line, in the presence of a magnetic field. An AT line is predicted in the RSB picture, but is argued not to exist according to the droplet picture. The existence of the AT line is arguably the most striking features of the immensely complicated mean-field-theory of spin glasses¹, and so it is of intrinsic interest to know whether or not it occurs in real spin glasses. Some time ago Bray and Roberts⁵ investigated whether or not there is an AT line below six dimensions (six is the upper critical dimension for spin glasses) using renormalization group ideas, but did not find a stable, accessible fixed point. More recently, Temesvari⁶ claimed to be able to follow the AT line down to just below six dimensions.

Theoretically, it is interesting to understand spin glasses in a range of dimensions. However, this is hard since one cannot simulate systems with a large number of spins N , where $N = L^d$, for a *range* of linear sizes L (needed to do finite-size scaling) if the dimension d is large. In particular, the the mean-field regime, $d > 6$, is not directly amenable to simulation. Hence, instead, we investigate a *one-dimensional* model in which the rms strength of the interactions fall off with a power σ of the distance. This model has the advantage of allowing one to study large (linear) sizes, Furthermore, it has been suggested that changing the value of σ is analogous to changing the value of the space dimension d in a short-range model. Consequently we can study models in both the mean-field and non-mean-field regimes. The nature

of the spin glass phase in long-range models was discussed recently by Moore⁷.

Most previous work⁸⁻¹² on one-dimensional, long-range spin glass models, used Ising spins. However, in this paper, we build on a recent work by us¹³ which showed that, in mean field theory, an AT line also occurs in m -component vector spins provided the magnetic field is random in direction. More precisely, we perform Monte Carlo simulations on the three-component (Heisenberg) spin glass in one dimension with long-range interactions in the presence of a random magnetic field. This work follows on from a recent paper¹⁴ where we considered the same model in zero-field.

The plan of this paper is as follows: in Sec. II we describe the model and the Monte Carlo method used to simulate it, in Sec. III we describe the results, and finally in Sec. IV we summarize our conclusions.

II. MODEL AND METHOD

The Hamiltonian we study is

$$\mathcal{H} = - \sum_{\langle i,j \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - \sum_i \mathbf{h}_i \cdot \mathbf{S}_i, \quad (1)$$

where the \mathbf{S}_i , $i = 1, 2, \dots, N$, are classical 3-component Heisenberg spins of unit length, and the interactions J_{ij} are independent random variables with zero mean and a variance which falls off with a power of the distance r_{ij} between the spins,

$$[J_{ij}^2]_{av} \propto \frac{1}{r_{ij}^{2\sigma}}. \quad (2)$$

The notation $[\dots]_{av}$ indicates an average over the quenched disorder. In addition we set $J_{ii} = 0$. The magnetic fields h_i^μ , where μ denotes a cartesian spin component, are chosen to be independent Gaussian random fields, uncorrelated between sites, with zero mean, which

satisfy

$$[h_i^\mu h_j^\nu]_{\text{av}} = h_r^2 \delta_{ij} \delta_{\mu\nu}. \quad (3)$$

Following Leuzzi et. al.⁸, and continuing along the lines of the zero-field paper¹⁴, the interactions of our model are such that, instead of the *magnitude* of the interaction falling off with distance like Eq. (2), it is the *probability* of there being a non-zero interaction between sites (i, j) which falls off, and when an interaction does occur, its variance is independent of r_{ij} . The mean number of non-zero interactions from a site, which we call z , can be fixed, and here we take $z = 6$. To generate the set of pairs (i, j) that have an interaction with the desired probability we choose spin i randomly, and then choose j ($\neq i$) at distance r_{ij} with probability

$$p_{ij} = \frac{r_{ij}^{-2\sigma}}{\sum_{j (j \neq i)} r_{ij}^{-2\sigma}}, \quad (4)$$

where, for r_{ij} , we put the sites on a circle and use the distance of the chord, i.e.

$$r_{ij} = \frac{N}{\pi} \sin \left[\frac{\pi}{N} (i - j) \right]. \quad (5)$$

If i and j are already connected, we repeat the process until we find a pair which has not been connected before. We then connect i and j with an interaction picked from a Gaussian interaction whose mean is zero and whose standard deviation is J , which we set equal to 1. This process is repeated precisely $N_b = zN/2$ times.

The result is that each pair (i, j) will be connected with a probability P_{ij} which must satisfy the condition $N \sum_j P_{ij} = Nz$ since P_{ij} only depends on $|i - j|$, $P_{ii} = 0$, and there are precisely $Nz/2$ connected pairs. It follows that, for a fixed site i ,

$$\sum_j [J_{ij}^2]_{\text{av}} = J^2 \sum_j P_{ij} = J^2 z. \quad (6)$$

The mean-field spin glass transition temperature for m -component vector spins is given, for zero field, by¹⁵

$$T_c^{MF} = \frac{1}{m} \left(\sum_j [J_{ij}^2]_{\text{av}} \right)^{1/2} = \frac{\sqrt{z}}{m} J, \quad (7)$$

where the last equality follows from Eq. (6). In mean-field theory, the critical magnetic field at zero-temperature for m -component vector spins¹³ is (after accounting for the different normalization of the spins in Ref. [13]) given by

$$\begin{aligned} h_c^{MF} &\equiv h_{AT}^{MF}(T=0) = \sqrt{\frac{m}{m-2}} T_c^{MF} \\ &= \sqrt{\frac{z}{m(m-2)}} J, \end{aligned} \quad (8)$$

We set $J = 1$ so that, for the situation here,

$$J = 1, \quad z = 6, \quad m = 3, \quad (9)$$

we have

$$T_c^{MF} = \frac{\sqrt{6}}{3} \simeq 0.816, \quad (10)$$

the same as for the nearest-neighbor Heisenberg spin glass on a simple cubic lattice, and

$$h_c^{MF} = \sqrt{2} \simeq 1.414. \quad (11)$$

The ratio of these two quantities, which we shall refer to later, is given by

$$\frac{h_c^{MF}}{T_c^{MF}} = \sqrt{3} \simeq 1.732. \quad (12)$$

According to Ref. [13], a good approximation for the AT line in the mean field theory of the Heisenberg ($m = 3$) spin glass is

$$\frac{h_{AT}^{MF}(T)}{T_c^{MF}} = \left(\frac{4m}{m+2} t^3 \right)^{1/2} \quad (13)$$

where

$$t = \frac{T - T_c^{MF}}{T_c^{MF}}. \quad (14)$$

In Eq. (13) we have again allowed for the different normalization of the spins in Ref. [13]. Equation (13) is exact, in mean field theory, near T_c^{MF} , and, for $m = 3$, works very well down to quite low temperatures, see Fig. 1 in Ref. [13]. Even for $T = 0$, ($t = 1$), Eq. (13) gives $h_c^{MF}/T_c^{MF} = \sqrt{12/5} \simeq 1.549$, whereas the correct value in mean field theory is, according to Eq. (12), $h_c^{MF}/T_c^{MF} \simeq 1.732$.

We perform Monte Carlo simulations for this model in a magnetic field for three values of σ : 0.6, 0.75, and 0.85. As discussed in the zero-field paper¹⁴, 0.6 lies in the mean-field regime, while the other two values of σ are in the non-mean-field regime.

An approximate connection between a value of σ and the effective dimension of an equivalent short-range model, d_{eff} , is¹⁶

$$d_{\text{eff}} = \frac{2}{2\sigma - 1}. \quad (15)$$

In the non mean-field region, a more accurate connection, which involves the exponent η_{SR} of the short-range model, can also be obtained¹⁶, but we will neglect this correction here since we have little information on η_{SR} . From Eq. (15) the effective dimensions corresponding to $\sigma = 0.75$ and 0.85 are $d_{\text{eff}} = 4$ and $d_{\text{eff}} \simeq 3$, respectively.

We continue to use the technology described in the zero-field paper¹⁴: overrelaxation sweeps, heatbath

sweeps, and parallel tempering. We perform one heatbath sweep, and one parallel tempering sweep for every ten overrelaxation sweeps.

The Gaussian nature of the interactions and the magnetic fields affords a useful test for equilibration. The relation

$$U = \frac{J^2}{T} \frac{z}{2} (q_l - q_s) + \frac{h_r^2}{T} (q - 1), \quad (16)$$

is valid in equilibrium but, very plausibly, the two sides approach their common equilibrium value from opposite directions as equilibrium is approached. Here

$$U = -\frac{1}{N} \left[\sum_{\langle i,j \rangle} \epsilon_{ij} J_{ij} \langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle + \sum_{i,\mu} h_i^\mu \langle S_i^\mu \rangle \right]_{\text{av}} \quad (17)$$

is the average energy per spin, $q = (1/N) \sum_i [\langle \mathbf{S}_i \rangle \cdot \langle \mathbf{S}_i \rangle]_{\text{av}}$ is the Edwards-Anderson order parameter, $q_l = (1/N_b) \sum_{\langle i,j \rangle} \epsilon_{ij} [\langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle^2]_{\text{av}}$ is the “link overlap”, and $q_s = (1/N_b) \sum_{\langle i,j \rangle} \epsilon_{ij} [\langle (\mathbf{S}_i \cdot \mathbf{S}_j)^2 \rangle]_{\text{av}}$, where $N_b = zN/2$, and $\epsilon_{ij} = 1$ if there is a bond between i and j and is zero otherwise. Equation (16) is easily derived by integrating by parts Eq. (17) with respect to J_{ij} and h_i^μ since they have Gaussian distributions.

We determine both sides of Eq. (16) for different numbers of Monte Carlo sweeps (MCS) which increase in a logarithmic manner, each value being twice the previous one. In all cases we average over the last half of the sweeps. We consider the data to be equilibrated, if, when averaging over a large number of samples, Eq. (16) is satisfied for at least the last two points. Note that in the numerics we set $J = 1$. Table I lists the parameters of the simulation.

We determine the wave-vector-dependent spin-glass susceptibility, given by¹³

$$\chi_{\text{SG}}(k) = \frac{1}{N} \sum_{i,j} \frac{1}{m} \sum_{\mu,\nu} \left[(\chi_{ij}^{\mu\nu})^2 \right]_{\text{av}} e^{ik(i-j)}, \quad (18a)$$

where

$$\chi_{ij}^{\mu\nu} = \langle S_i^\mu S_j^\nu \rangle - \langle S_i^\mu \rangle \langle S_j^\nu \rangle, \quad (18b)$$

in which $\langle \dots \rangle$ denotes a thermal average and $[\dots]_{\text{av}}$ an average over the disorder. To avoid bias, each thermal average is obtained from a separate copy of the spins, so we simulate four copies at each temperature. The spin glass correlation length is then determined from

$$\xi_{\text{SG}} = \frac{1}{2 \sin(k_{\min}/2)} \left(\frac{\chi_{\text{SG}}(0)}{\chi_{\text{SG}}(k_{\min})} - 1 \right)^{1/(2\sigma-1)}, \quad (19)$$

where $k_{\min} = (2\pi/N)$.

According to finite-size scaling^{14,17}, the correlation length of the finite-system varies, near the transition temperature T_c , as

$$\frac{\xi}{N} = \mathcal{X}[N^{1/\nu}(T - T_c)], \quad (2/3 \leq \sigma < 1), \quad (20a)$$

$$\frac{\xi}{N^{1/3}} = \mathcal{X}[N^{1/3}(T - T_c)], \quad (1/2 < \sigma \leq 2/3), \quad (20b)$$

TABLE I: Parameters of the simulations. N_{samp} is the number of samples, N_{equil} is the number of overrelaxation Monte Carlo sweeps for equilibration for each of the $2N_T$ replicas for a single sample. The same number of sweeps is done in the measurement phase, with a measurement performed every four overrelaxation sweeps. The number of heatbath sweeps is equal to 10% of the number of overrelaxation sweeps. T_{\min} and T_{\max} are the lowest and highest temperatures simulated, and N_T is the number of temperatures used in the parallel tempering.

σ	h_r	N	N_{samp}	N_{equil}	T_{\min}	T_{\max}	N_T
0.6	0.1	128	6000	512	0.20	0.50	21
0.6	0.1	256	6000	512	0.20	0.50	23
0.6	0.1	512	6000	2048	0.20	0.50	24
0.6	0.1	1024	6000	4096	0.20	0.50	27
0.6	0.1	2048	6000	8192	0.20	0.50	30
0.6	0.1	4096	2000	16384	0.25	0.45	40
0.6	0.1	8192	1000	32768	0.27	0.45	42
0.6	0.1	16384	1000	16384	0.30	0.42	45
0.75	0.1	128	6000	8192	0.05	0.24	21
0.75	0.1	256	6000	16384	0.05	0.24	22
0.75	0.1	512	6000	16384	0.10	0.30	24
0.75	0.1	1024	3500	65536	0.10	0.24	20
0.75	0.1	2048	2300	131072	0.10	0.24	23
0.75	0.1	4096	1000	262144	0.10	0.30	45
0.85	0.05	128	10000	32768	0.015	0.20	25
0.85	0.05	256	9000	65536	0.015	0.20	34
0.85	0.05	512	5000	262144	0.02	0.20	38
0.85	0.05	1024	5000	524288	0.03	0.20	40
0.85	0.05	2048	4100	524288	0.05	0.20	34
0.85	0.1	128	6000	8192	0.05	0.25	21
0.85	0.1	256	2500	16384	0.05	0.25	22
0.85	0.1	512	2000	32768	0.05	0.25	24
0.85	0.1	1024	1200	262144	0.05	0.20	34
0.85	0.1	2048	1600	262144	0.05	0.20	40
0.85	0.1	4096	1800	131072	0.10	0.20	30

in which ν , the correlation length exponent, is given, in the mean-field regime, by $\nu = 1/(2\sigma - 1)$. It follows that, if there is a transition at $T = T_c$, data for ξ/N ($\xi/N^{1/3}$ in the mean-field region) for different system sizes N should cross at T_c .

We also present data for $\chi_{\text{SG}} \equiv \chi_{\text{SG}}(0)$, which has the finite-size scaling form

$$\frac{\chi_{\text{SG}}}{N^{2-\eta}} = \mathcal{C}[N^{1/\nu}(T - T_c)], \quad (2/3 \leq \sigma < 1), \quad (21a)$$

$$\frac{\chi_{\text{SG}}}{N^{1/3}} = \mathcal{C}[N^{1/3}(T - T_c)], \quad (1/2 < \sigma \leq 2/3). \quad (21b)$$

Hence curves of $\chi_{\text{SG}}/N^{2-\eta}$ ($\chi_{\text{SG}}/N^{1/3}$ in the mean-field regime) should also intersect. This is particularly useful for long-range models since η is given by the simple expression $2 - \eta = 2\sigma - 1$ *exactly*.

In practice, there are corrections to this finite-size-scaling, so data for different sizes do not all intersect at the exactly the same temperature. Including leading corrections to scaling, the intersection temperature

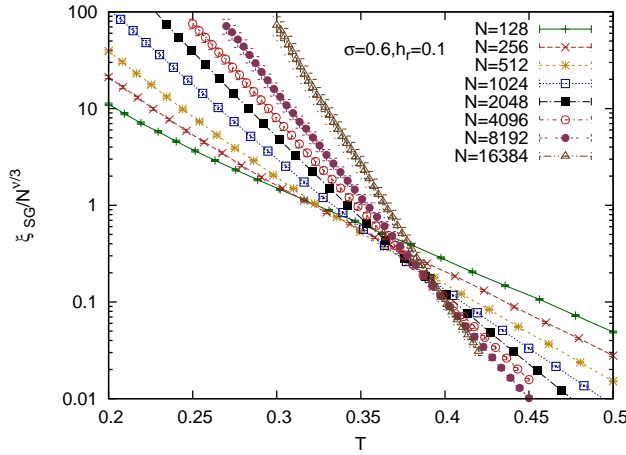


FIG. 1: (Color online) A finite size scaling plot of data for ξ_{SG} , for $\sigma = 0.6$ in a magnetic field of $h_r = 0.1$. The intersections indicate that there is a spin glass phase transition (AT line) in the presence of a magnetic field.

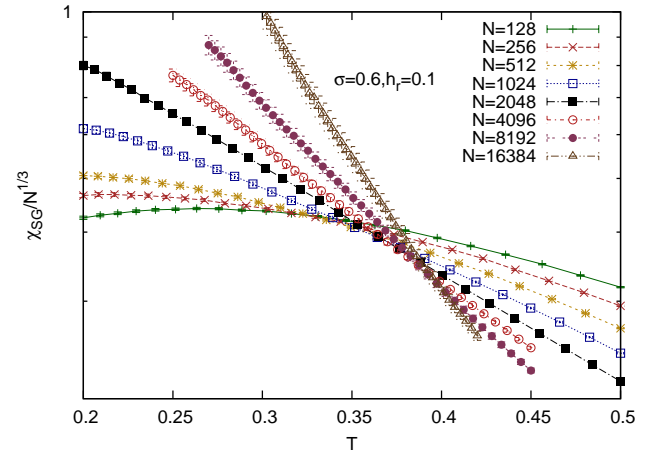


FIG. 2: (Color online) A finite size scaling figure of data for χ_{SG} , for $\sigma = 0.6$ with $h_r = 0.1$. As in Fig. 1, the intersections indicate the presence of an AT line.

$T^*(N, 2N)$ for sizes N and $2N$ varies as^{16,18–20}

$$T^*(N, 2N) = T_c + \frac{A}{N^\lambda}, \quad (22)$$

where A is the amplitude of the leading correction, and, in the non mean-field regime, the exponent λ is given by

$$\lambda = \frac{1}{\nu} + \omega \quad (23)$$

where ω is the leading correction to scaling exponent.

III. RESULTS AND ANALYSIS

A. $\sigma = 0.6$

We recall that $\sigma = 0.6$ lies in the mean-field regime. In this regime, simulations of the corresponding Ising model^{9,12} found an AT line. From our plots for the Heisenberg spin glass in Figs. 1 and 2 for $h_r = 0.1$, we come to the same conclusion here. According to Eqs. (20b) and (21b), data for $\xi_{SG}/N^{\nu/3}$ and $\chi_{SG}/N^{1/3}$ should intersect at the transition temperature. We do, indeed find intersections, though the intersection temperatures vary somewhat with size.

The intersection temperatures are shown in Fig. 3. Fitting the intersection temperatures to Eq. (22), using the known value^{14,16} $\lambda = 0.467$ we find $T_{SG} = 0.406 \pm 0.012$ from ξ_{SG} (omitting the two smallest sizes), and $T_{SG} = 0.405 \pm 0.007$ from χ_{SG} (including all the data). These two results agree well with each other. Note that the intersection temperatures *increase* with increasing size, which suggests that they will not disappear in the thermodynamic limit.

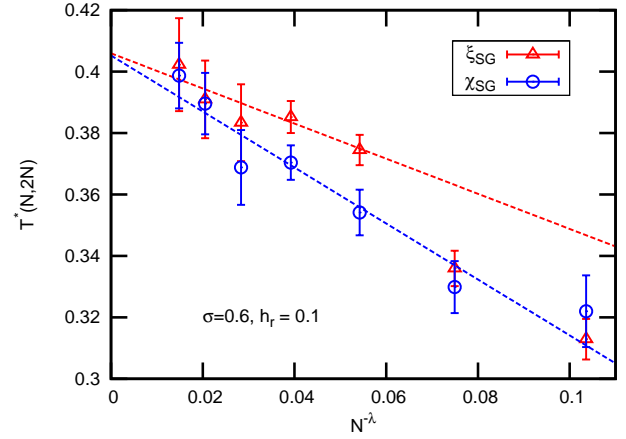


FIG. 3: (Color online) A plot of the intersection temperatures $T^*(N, 2N)$ for $\sigma = 0.6$, obtained from the data in Figs. 1 and 2, as a function of $N^{-\lambda}$, with λ fixed to its value 0.467, which is known exactly because $\sigma = 0.6$ is in the mean-field regime. The fits give: $T_{SG} = 0.406 \pm 0.012$ from ξ_{SG} and $T_{SG} = 0.405 \pm 0.007$ from χ_{SG} . For comparison, the zero field transition temperature is¹⁴ $T_{SG} \simeq 0.563$.

It is interesting to compare this point on the AT line, $(T, h_{AT}(T)) = (0.405, 0.1)$, with mean field predictions. Replacing T_c^{MF} with the actual zero field transition temperature of¹⁴ $T_c = 0.563$ in Eqs. (13) and (14), we find that $T = 0.405$ gives a field $h_{AT}(T = 0.405) = 0.130$, which is slightly larger than the actual field value of 0.1. Hence the value of the field on the AT line for $\sigma = 0.6$ is *somewhat* less than that expected in mean field theory, even allowing for the reduction in T_c from its mean field value.

To conclude this section, the data suggests that there

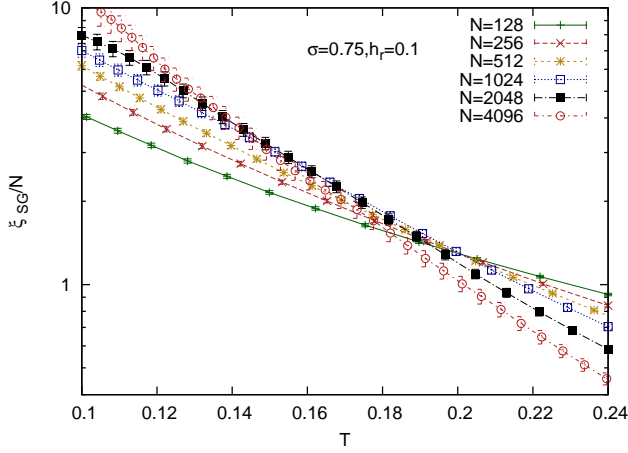


FIG. 4: (Color online) A finite size scaling plot of data for ξ_{SG} for $\sigma = 0.75$. The magnetic field is $h_r = 0.1$.

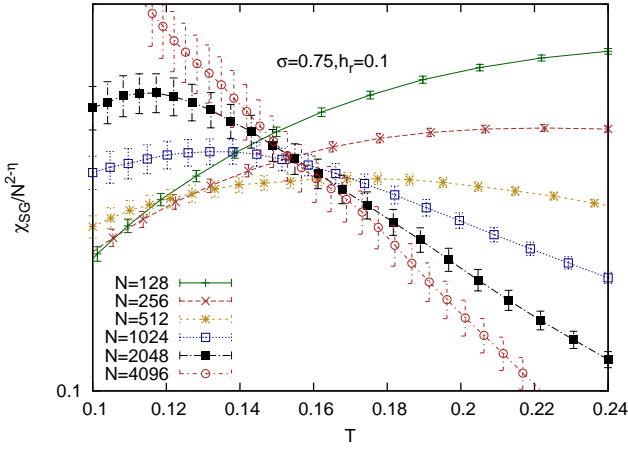


FIG. 5: (Color online) A finite size scaling plot of data for χ_{SG} for $\sigma = 0.75$. The magnetic field is $h_r = 0.1$.

is an AT line for $\sigma = 0.6$, as found earlier^{9,12} for the corresponding Ising model.

B. $\sigma = 0.75$

According to Eq. (15) (which is approximate) this value of σ corresponds to a short-range model in $d = 4$ dimensions. For the corresponding Ising study, Ref. [12] did not find an AT line, though this conclusion was subsequently challenged in Ref. [9].

Data for the spin glass correlation length and susceptibility are shown in Figs. 4 and 5. There are clearly intersections. Since we are now in the non mean-field regime, the exponent λ in Eq. (22) is not known and so should be treated as a fit parameter. However, the intersection temperatures have quite large error bars, and

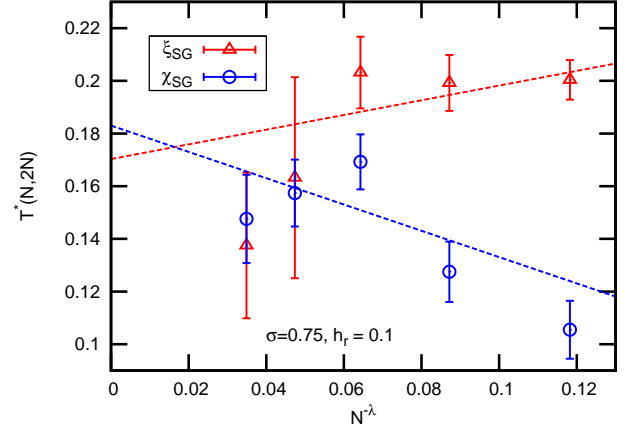


FIG. 6: (Color online) A plot of the intersection temperatures $T^*(N, 2N)$ for $\sigma = 0.75$, obtained from the data in Figs. 4 and 5, as a function of $N^{-\lambda}$ with $\lambda = 0.44$, the value found for $\sigma = 0.75$ in our zero field study¹⁴. We were not able to determine λ with any degree of precision from the data in Figs. 4 and 5. Both sets of data are compatible with a transition temperature of about 0.16 though there is also a hint in the data that the intersection temperatures start to drop for the largest sizes. For comparison, the zero field transition temperature is¹⁴ $T_c \simeq 0.357$.

do not seem to vary monotonically, as shown in Fig. 6. Thus the data is not of good enough quality to determine λ , and, in the plot, we have, rather arbitrarily used the value $\lambda = 0.44$ obtained for $\sigma = 0.75$ in our zero-field study¹⁴.

Both sets of data are compatible with a non-zero T_c of about 0.16, which is to be compared with the zero field spin glass transition temperature $T_{SG} \simeq 0.357$. However, there is a suggestion in the data that the values of T^* decrease for the largest pairs of sizes, so perhaps one should treat this conclusion with some caution.

As we did above for $\sigma = 0.6$, we compare the putative point on the AT line, $(T, h_{AT}(T)) = (0.16, 0.1)$, with mean field predictions. Replacing T_c^{MF} with the actual zero field transition temperature of¹⁴ $T_c = 0.357$ in Eqs. (13) and (14), we find that $T = 0.16$ gives a field $h_{AT}(T = 0.16) = 0.227$, which is considerably larger than the actual field value of 0.1. Hence, if the intersections in Figs. 4 and 5, do represent a transition in a field, the value of this AT field is *considerably* less than that expected in mean field theory, even allowing for the reduction in T_c from its mean field value.

C. $\sigma = 0.85$

According to Eq. (15), $\sigma = 0.85$ corresponds to a short-range model in close to three dimensions. In their study of the Ising version of this model, Ref. [12] did not find an AT line for this value of σ . Ref. [9] did not consider

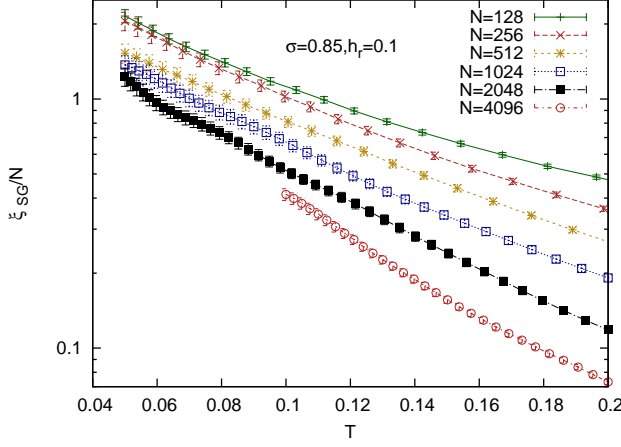


FIG. 7: (Color online) A finite size scaling plot of data for ξ_{SG} for $\sigma = 0.85$. The magnetic field is $h_r = 0.1$. No intersections are found for the range of temperatures that we simulate indicating that there is no phase transition in this range. For comparison, the zero field transition temperature is¹⁴ $T_c \simeq 0.165$.

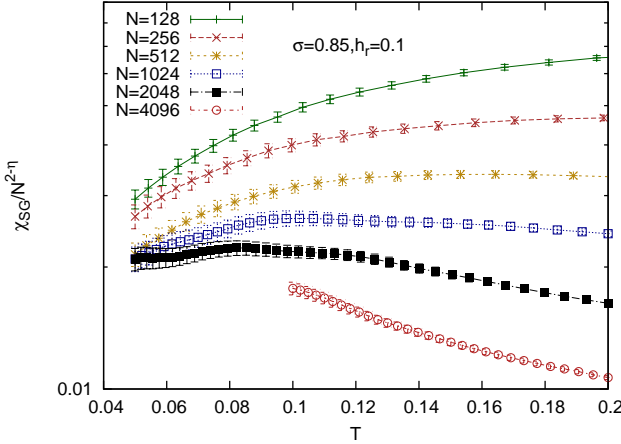


FIG. 8: (Color online) A finite size scaling plot of data for χ_{SG} for $\sigma = 0.85$. The magnetic field is $h_r = 0.1$. At $T \simeq 0.05$, the largest sizes for which we have data, $N = 512, 1024$ and 2048 merge together. For comparison, the zero field transition temperature is¹⁴ $T_c \simeq 0.165$.

this value of σ .

We show finite-size scaling plots for ξ_{SG} and χ_{SG} for $h_r = 0.1$ and 0.05 in Figs. 7–10. For $h_r = 0.1$, the ξ_{SG} data does not intersect down to the lowest temperature, 0.05 , which is to be compared with the zero field transition temperature¹⁴ $T_c \simeq 0.165$. The χ_{SG} data for $h_r = 0.1$ seem to merge at the lowest temperature for the largest sizes that could be studied in this region.

For $h_r = 0.05$, the data for *both* ξ_{SG} and χ_{SG} merge at the lowest temperatures. One possible explanation of this is that the critical field at $T = 0$ is $h_c \lesssim 0.05$.

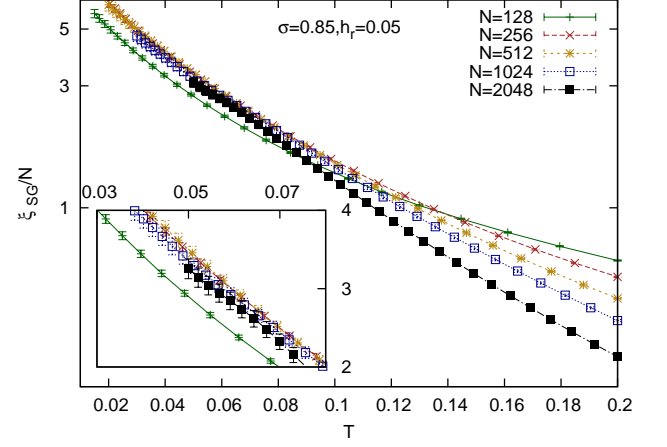


FIG. 9: (Color online) A finite size scaling plot of data for ξ_{SG} for $\sigma = 0.85$. The magnetic field is $h_r = 0.05$. The data appears to merge at the lowest temperatures. The inset shows an enlargement of a region of the data.

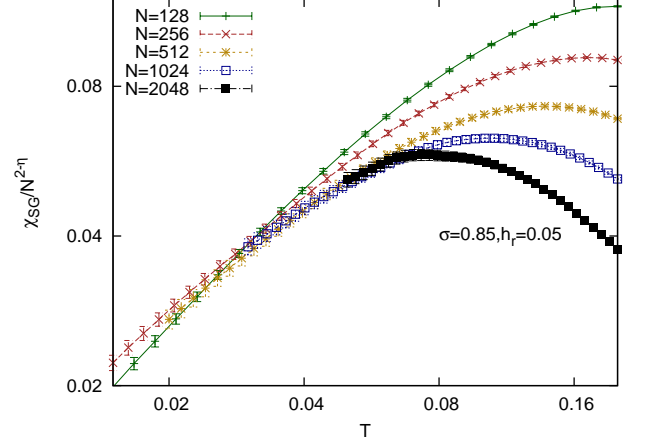


FIG. 10: (Color online) A log-log finite size scaling plot of data for χ_{SG} for $\sigma = 0.85$. The magnetic field is $h_r = 0.05$. The data appears to merge at the lowest temperatures.

Another possibility is that, for this small field, we are in a crossover region between zero-field behavior, where there is a transition, to finite-field behavior where there is none.

According to Eq. (12), in mean field theory the zero-temperature critical field, h_c is 1.732 times the zero-field transition temperature T_c . If we assume, based on our data, that $h_c \lesssim 0.05$ then, using $T_c \simeq 0.165$ from Ref. [14], we have a ratio $h_c/T_c \lesssim 0.30$, which is about 17% of the mean field result, i.e. *considerably* smaller. Hence, if the data in Figs. 9 and 10 is interpreted to show a critical field of around 0.05 , then this is *much* smaller than in mean field theory, even allowing for the (substantial) reduction in the zero-field T_c relative to the

mean field prediction.

IV. CONCLUSION

We have studied existence or otherwise of the AT line in the 1-dimensional Heisenberg spin glass with interactions which fall off as a power of the distance. We are able to study a large range of sizes in the temperature range of interest: $N \leq 16384$ for $\sigma = 0.6$, $N \leq 4096$ for $\sigma = 0.75$, and $N \leq 2048$ for $\sigma = 0.85$ (up to 4096 at somewhat higher temperatures).

For $\sigma = 0.6$, which is in the mean-field regime ($\sigma < 2/3$), we find an AT line. For $\sigma = 0.75$, which is in the non-mean-field regime and corresponds to a short-range model with dimension about 4, the data does appear to find a phase transition for a field $h_r = 0.1$, though the intersection temperatures in the finite-size scaling plots drop for the largest sizes, which might give one pause to accept that conclusion with certainty. For $\sigma = 0.85$, which corresponds to a dimension of about 3, the data for $h_r = 0.05$ can be interpreted as indicating that this value of field is close to a critical AT field in the limit of zero temperature. However, it can also be interpreted as indicating a crossover between the zero-field transition and behavior in a field which has no transition.

For the corresponding Ising model, Ref. [12] finds an

AT line in the mean-field regime but not in the non-mean-field regime. This conclusion was challenged in Ref. [9], who claim that there is also an AT line in the non-mean-field regime, at least up for σ between $2/3$ and 0.75 . A motivation for the present study was to see if a clearer numerical picture can emerge from the Heisenberg spin glass, where it is possible to study larger sizes than for the Ising case. Unfortunately, it seems that corrections to scaling are quite large in the Heisenberg case, so we are not able to give a precise value for the lower critical dimension of the AT line from the data in our paper. If there is no AT line, the system breaks up into domains of size ℓ (Imry-Ma length) which can be large at low temperatures. A possible explanation of our results is therefore that $\ell(T \rightarrow 0)$ for $\sigma = 0.75, h_r = 0.1$ is larger than the largest system size, $N = 4096$, and that for $\sigma = 0.85, h_r = 0.05$, $\ell(T \rightarrow 0)$ is about equal to the largest system size at the lowest temperature, $N = 2048$.

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